

## The Independent or "Student's" t-ratio

The t-ratio is a powerful statistic that allows a user to determine if a pair of sample means from two presumed-to-be independent groups are sufficiently mathematically different to conclude that they are drawn from two distinct populations; or, contrariwise, due to a small or negligible difference one must conclude that they come from the same population.

When a comparison of means is made with the t statistic and it yields a sufficiently large result (relative to other factors in the t equation such as sample size) the answer will be described as "statistically significant" at a particular probability level. This will tell us that the means of the two groups are SO DIFFERENT that we must conclude that they (the groups) are indeed independent of each other--they are, in other words, drawn from two different worlds. This would lead us to REJECT the Null Hypothesis for the t test which holds that the two means equal each other.

When the means of both groups are identical or nearly so this will yield a small t statistic and we will conclude that our two presumed-to-be independent groups really come from one world (i.e., one population)...they are, statistically speaking, from the same universe. Such a result would lead us to ACCEPT the Null Hypothesis for the t test, i.e., ACCEPT the hypothesis which holds that the two means essentially equal each other.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left[ \frac{\sum X_1^2 - \frac{(\sum X_1)^2}{N_1} + \sum X_2^2 - \frac{(\sum X_2)^2}{N_2}}{(N_1 + N_2) - 2} \right] \cdot \left[ \frac{N_1 + N_2}{N_1 \cdot N_2} \right]}}$$

where  $\bar{X}_1$  = the mean of the first group of scores

$\bar{X}_2$  = the mean of the second group of scores

$\sum X_1^2$  = the sum of the squared score values of the first group

$\sum X_2^2$  = the sum of the squared score values of the second group

$(\sum X_1)^2$  = the square of the sum of the scores in the first group

$(\sum X_2)^2$  = the square of the sum of the scores in the second group

$N_1$  = the number of scores in the first group

$N_2$  = the number of scores in the second group

# Independent t test

## When to use the Independent t statistic

Dr. Anthony A. Walsh, Basic Statistics, Fall, 1996

- ...use this test in an experimental situation where you have the typical research design involving an EXPERIMENTAL and a CONTROL group, and the EXPERIMENTAL group has been exposed to an independent variable and the control group has not...and you wish to test to see if the independent variable has had any effect; and,
- ...use this test in an alternate type of research situation where you have two groups of presumed to be independent subjects and you wish to test for their independence on some variable measured at the interval or ratio level, e.g., MEN vs. WOMEN on a mechanical aptitude test; 5 year olds vs. 12 year olds on a math comprehension test; freshman vs. sophomores on a measure of language comprehension, etc.

Note: the INDEPENDENT t STATISTIC, also known as the t RATIO or STUDENT'S t TEST does not require that your two groups are equal in number(size), although from the perspective of research design, you should always try to have equal Ns, or as nearly equal as possible.

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# Requirements for the t test

- The samples must be random
- The measured traits must not depart significantly from normality in the population
- The sample SDs must be fairly similar
- The samples must be independent of each other
- The data must be interval or ratio
- Comparisons must be on measures of the SAME trait

# Independent t Ratio

## Formula

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left[ \frac{\sum X_1^2 - \frac{(\sum X_1)^2}{N_1} + \sum X_2^2 - \frac{(\sum X_2)^2}{N_2}}{(N_1 + N_2) - 2} \right] \cdot \left[ \frac{N_1 + N_2}{N_1 \cdot N_2} \right]}}$$

Note:  $N_1 \cdot N_2$  in this formula means MULTIPLY the number of cases in the  $X_1$  group by the number of cases in the  $X_2$  group EVEN IF THE TWO NUMBERS ARE NOT EQUAL TO EACH OTHER.

Remember: the Independent t statistic DOES NOT REQUIRE EQUAL Ns.

# INDEPENDENT t TEST

## The Null Hypothesis & t

- The NULL HYPOTHESIS may be expressed differently depending on a researcher's circumstances. The tests of Null with t are described as being either

- TWO-TAIL TESTS ... or ...  
ONE-TAIL TESTS ... of the t ratio.

- ...TWO-TAIL TEST - in the two-tail test situation, the researcher believes that the results may favor larger values for Mean 1 or Mean 2 but he does not know which. Therefore,  $H_0: M1 = M2$

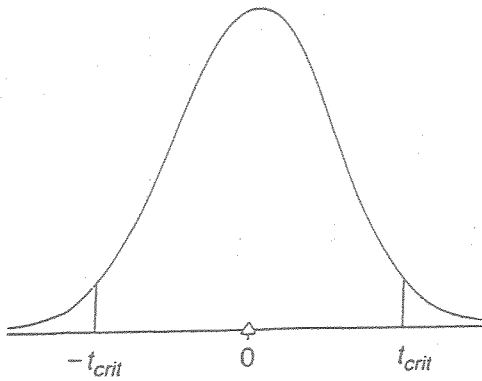
IF  $Halt:$   $\overline{X}_1 \neq \overline{X}_2$   
THEN  $H_0:$   $\overline{X}_1 = \overline{X}_2$

- ONE-TAIL TEST - in the one-tail test, the researcher predicts which mean will be larger, Mean 1 or Mean 2. If, for example, the researcher predicts that Mean 1 will be larger than Mean 2, then  $H_0: M1 \geq M2$

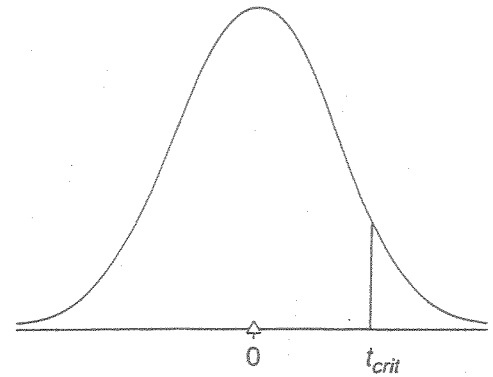
IF  $Halt:$   $\overline{X}_1 > \overline{X}_2$   
THEN  $H_0:$   $\overline{X}_1 \leq \overline{X}_2$

Note:  $M1$  stands for mean 1, normally written as capital X with a bar over it.  
 $M2$  stands for mean 2, normally written as capital X with a bar over it.

CRITICAL VALUES OF  $t$



Two-tailed or Nondirectional Test  
LEVEL OF SIGNIFICANCE  
( $p$ -value in color)



One-tailed or Directional Test  
LEVEL OF SIGNIFICANCE  
( $p$ -value in color)

$df$	.05	.01	.001	$df$	.05	.01	.001
1	12.706	63.657	636.62	1	6.314	31.821	318.31
2	4.303	9.925	31.598	2	2.920	6.965	22.326
3	3.182	5.841	12.924	3	2.353	4.541	10.213
4	2.776	4.604	8.610	4	2.132	3.747	7.173
5	2.571	4.032	6.869	5	2.015	3.365	5.893
6	2.447	3.707	5.959	6	1.943	3.143	5.208
7	2.365	3.499	5.408	7	1.895	2.998	4.785
8	2.306	3.355	5.041	8	1.860	2.896	4.501
9	2.262	3.250	4.781	9	1.833	2.821	4.297
10	2.228	3.169	4.587	10	1.812	2.764	4.144
11	2.201	3.106	4.437	11	1.796	2.718	4.025
12	2.179	3.055	4.318	12	1.782	2.681	3.930
13	2.160	3.012	4.221	13	1.771	2.650	3.852
14	2.145	2.977	4.140	14	1.761	2.624	3.787
15	2.131	2.947	4.073	15	1.753	2.602	3.733
16	2.120	2.921	4.015	16	1.746	2.583	3.686
17	2.110	2.898	3.965	17	1.740	2.567	3.646
18	2.101	2.878	3.922	18	1.734	2.552	3.610
19	2.093	2.861	3.883	19	1.729	2.539	3.579
20	2.086	2.845	3.850	20	1.725	2.528	3.552
21	2.080	2.831	3.819	21	1.721	2.518	3.527
22	2.074	2.819	3.792	22	1.717	2.508	3.505
23	2.069	2.807	3.767	23	1.714	2.500	3.485
24	2.064	2.797	3.745	24	1.711	2.492	3.467
25	2.060	2.787	3.725	25	1.708	2.485	3.450
26	2.056	2.779	3.707	26	1.706	2.479	3.435
27	2.052	2.771	3.690	27	1.703	2.473	3.421
28	2.048	2.763	3.674	28	1.701	2.467	3.408
29	2.045	2.756	3.659	29	1.699	2.462	3.396
30	2.042	2.750	3.646	30	1.697	2.457	3.385
40	2.021	2.704	3.551	40	1.684	2.423	3.307
60	2.000	2.660	3.460	60	1.671	2.390	3.232
120	1.980	2.617	3.373	120	1.658	2.358	3.160
$\infty$	1.960	2.576	3.291	$\infty$	1.645	2.326	3.090